Explicit Approximations to the Solution of Colebrook's Friction Factor Equation

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The friction factor equation developed by Colebrook (1939) for tubulent

$$\frac{1}{\sqrt{f_D}} = -2.0 \log \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f_D}}\right) \tag{1}$$

flow has received wide acceptance, probably because it was used by Moody (1944) in the preparation of his friction factor charts. However, Colebrook's equation is implicit in Darcy's friction factor, f_D, and must therefore be solved by iteration, a formidable task in 1944 when Moody presented his charts which, no doubt, accounts for their popularity. Solution of Eq. 1 by numerical methods to any desired degree of precision is accomplished easily, quickly and cheaply with today's digital computers.

Moody (1947) presented an explicit friction equation applicable to the

$$f_D = 5.5 \times 10^{-3}\left[1 + \left(2 \times 10^4\,\epsilon/D + \frac{10^6}{\mathrm{Re}}\right)^{1/3}\right] \tag{2}$$

turbulent region of the flow. Equation 2 was said to yield friction factors within ±5% of those of Eq. 1 over the range $4{,}000 \leq \mathrm{Re} \leq 10^7$ and the relatively narrow range $0 \leq \epsilon/D \leq 0.01$. Later we show that a maximum absolute deviation of 15.9% occurs for this equation if the maximum value for ϵ/D is extended to 0.05.

A relationship developed by Wood (1966) for computer application gives the Darcy friction factor explicitly as

$$f_D = 0.094\left(\frac{\epsilon}{D}\right)^{0.225} + 0.53\left(\frac{\epsilon}{D}\right) + 88\left(\frac{\epsilon}{D}\right)^{0.4}(\mathrm{Re})^{-1.62\left(\frac{\epsilon}{D}\right)^{0.134}} \tag{3}$$

Our work shows that Wood's equation has a maximum absolute deviation of 6.0% over the ranges $4{,}000 < \mathrm{Re} < 10^7$ and $0.00004 < \epsilon/D < 0.05$.

Jain (1976) used the theoretical equation of Von Karmen and Prandtl for rough pipes

$$\frac{1}{\sqrt{f_D}} = 1.14 - 2 \log \frac{\epsilon}{D} \tag{4}$$

with curve fitting to yield

$$\frac{1}{\sqrt{f_D}} = 1.14 - 2 \log \left(\frac{\epsilon}{D} + \frac{21.25}{\mathrm{Re}^{0.9}}\right) \tag{5}$$

Equation 5 is said to differ from Eq. 1 no more than 1% over the ranges $5{,}000 < \mathrm{Re} < 10^7$ and $0.00004 < \epsilon/D < 5 \times 10^{-2}$.

Churchill (1977) developed an explicit equation said to be applicable to all values of Reynold's number and ϵ/D,

$$f = 8\left[\left(\frac{8}{\mathrm{Re}}\right)^{12} + \frac{1}{(A + B)^{3/2}}\right]^{1/12} \tag{6}$$

where

$$A = \left[2.457\, ln\left\{\left(\frac{7}{\mathrm{Re}}\right)^{0.9} + 0.27\,\frac{\epsilon}{D}\right\}\right]^{16} \tag{7}$$

and

$$B = \left(\frac{37530}{\mathrm{Re}}\right)^{16} \tag{8}$$

Chen (1979) presented an explicit equation which is superior to those of Moody, Wood, Jain and Churchill when compared to an iterative solution of Colebrook's equation.

$$\frac{1}{\sqrt{f_D}} = -2.0 \log\left[\frac{\epsilon/D}{3.7065} - \frac{5.0452}{\mathrm{Re}} \log\left(\frac{(\epsilon/D)^{1.1098}}{2.8257} + \frac{5.8506}{\mathrm{Re}^{0.8981}}\right)\right] \tag{9}$$

Equation 9 is also said to be good for all values of Re and ϵ/D. However it is compared with the equations of Colebrook, Wood and Churchill only over the ranges $4{,}000 < \mathrm{Re} < 4 \times 10^8$ and $5 \times 10^{-7} < \epsilon/D < 0.05$.

Our purpose in this paper is to present two additional explicit approximations to the solution of Colebrook's implicit equation. One of these is easier to use but less precise than Eq. 9 relative to the iterative solution of Eq. 1 while the second equation is both more complex and more precise than Eq. 9.

The turbulent portion of Moody's chart includes $4{,}000 \leq \mathrm{Re} \leq 10^8$ and $10^{-5} \leq \epsilon/D \leq 0.05$ with a resulting range for Darcy's friction factor of $0.001 \leq f_D \leq 0.077$. Using an average value of 0.04 for f_D gives a value for the term $2.5226/\sqrt{f_D}$ of about 13. Combining this value with Eq. 1 gives

$$\frac{1}{\sqrt{f_D}} = -2.0 \log\left(\frac{\epsilon/D}{3.7} + \frac{13}{\mathrm{Re}}\right) \tag{10}$$

Combining Eqs. 1 and 10 yields

$$\frac{1}{\sqrt{f_D}} = -2.0 \log\left[\frac{\epsilon/D}{3.7} - \frac{5.02}{\mathrm{Re}} \log\left(\frac{\epsilon/D}{3.7} + \frac{13}{\mathrm{Re}}\right)\right] \tag{11}$$

Equations 10 and 11 constitute explicit approximations for Eq. 1. Equation 11 is actually the first iteration in the numerical solution of Eq. 1. The constant 13 in Eq. 10, which was selected on the basis of an average value for f_D of 0.04, proves to be very nearly optimum over the range of interest and is much better value than is required by an iterative solution. Finally, Eq. 11 can be combined with Eq. 1 to give

$$\frac{1}{\sqrt{f_D}} = -2.0 \log \times\left\{\frac{\epsilon/D}{3.7} - \frac{5.02}{\mathrm{Re}} \log\left[\frac{\epsilon/D}{3.7} - \frac{5.02}{\mathrm{Re}} \log\left(\frac{\epsilon/D}{3.7} + \frac{13}{\mathrm{Re}}\right)\right]\right\} \tag{12}$$

Equation 12 constitutes the second iteration in the numerical solution of Eq. 1. It's maximum deviation from the numerical solution of Eq. 1 is only 0.11%.

A numerical comparison of Eqs. 2, 3, 5, 6, 9, 11, and 12 with the

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TABLE 1. COMPARISON OF EXPLICIT APPROXIMATIONS TO COLEBROOK'S FRICTION FACTOR EQUATION

	Moody Eq. 2	Wood Eq. 3	Jain Eq. 5	Churchill Eq. 6	Chen Eq. 9	Zigrang & Sylvester Eq. 11	Zigrang & Sylvester Eq. 12
Average Absolute Deviation, %	4.3	2.7	0.68	0.65	0.11	0.15	0.017
Maximum Absolute Deviation, %	16.0	6.0	3.10	3.10	0.32	0.95	0.110

numerical solution of Eq. 1 was conducted. A matrix of 60 test points was formed combining each of 10 roughness ratios with six different values for Reynold's number. The roughness ratios were 4×10^{-5}, 5×10^{-5}, 2×10^{-4}, 6×10^{-4}, 1.5×10^{-3}, 4×10^{-3}, 8×10^{-3}, 1.5×10^{-2}, 3×10^{-2} and 5×10^{-2}. The Reynold's numbers were 4×10^3, 3×10^4, 10^5, 10^6, 10^7 and 10^8. The absolute deviations relative to Colebrooks' equation were computed from

$$E = \left|\frac{f_D - f_{DC}}{f_{DC}}\right| \qquad (13)$$

and accumulated over the sixty points calculated for each of the seven explicit equations. The results are shown in Table 1.

Although each of the explicit approximations given in Eqs. 9, 11 and 12 is adequate for computational purposes, Eqs. 11 and 12 are recommended. Equation 9 requires more effort than Eq. 11 but less effort than Eq. 12. Likewise, Eq. 9 is more precise than Eq. 11 but less precise than Eq. 12. Consequently, Eq. 11 is recommended for use with hand-held calculators because it is relatively simple for its degree of precision with respect to the Colebrook equation equation. Clearly, Eq. 12 should be used with programmable calculators and digital computers.

NOTATION

- D = inside diameter of pipe
- E = Error, defined by Eq. 13
- ϵ = Roughness height
- f_D = Darcy friction factor
- f_{DC} = Darcy friction factor calculated from the Colebrook equation
- Re = Reynolds number

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Direct Contact Heat Transfer with Change of Phase: Theoretical Model

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The economical water desalination technique has led to increased interest in research on direct-contact heat transfer between two immiscible liquids. The mechanism of heat transfer during the course of vaporization of single liquid drop in an immiscible liquid medium has not yet been clarified.

Sideman and Isenberg (1967) presented a model about the quasi-steady state heat transfer, outside a two-phase bubble moving in a potential flow field, supposing that the heat flows into the bubble exclusively through the liquid-liquid interface situated at the bottom. The unsatisfactory agreement between the theoretical and experimental values of these authors is due to the assumption of having neglegible inside thermal resistance of the bubble.

The rigid sphere model of Tochitani et al. (1977) agrees quantitatively with their experimental results. This gives a more reasonable value of heat transfer coefficients than those obtained by the expression given by Sideman and Taitel (1964). The higher value of heat transfer coefficients given by the rigid sphere model during early stages of evaporation is ascribed to the predominant resistance prevailing inside the two-phase bubble. It is thus clear that the previous authors have not considered the proper configuration of liquid-liquid interface for transfer of heat.

The aim of this paper is to develop a theoretical model based on actual liquid-liquid heat transfer interface, by introducing the effect of viscous shear on the spreading of dispersed liquid over the bubble surface.

Considering only the surface and interfacial free energies, Mori (1978) obtained the following relation for the film spreading coefficient in terms of interfacial tensions:

$$S_d = \sigma_c - (\sigma_d + \sigma_{dc})$$

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